Interpretation of Drag Reduction Phenomenon in Wavy Films of Polymer Solutions

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INTRODUCTION

A few reports (e.g., Kale et al., 1973; Mashelkar and Devarajan, 1976), which indicate that drag reduction under laminar conditions is possible, have appeared in the past. Popadic's paper (1975) pertaining to drag reduction in wavy laminar film flows of polymer solutions is another example. Although the experimental results are convincing, the mechanistic interpretation of the phenomenon proposed by Popadic appears to be unsound. In this paper, we shall show why it is so, and shall propose and test an alternative hypothesis which appears to be more rational.

POPADIC'S HYPOTHESIS

Focussing on the stretching deformations present in secondary flows in wavy film flows, Popadic argued that the interaction of elasticity and these stretching motions would be the same as that in the case of turbulent flow of polymer solutions leading to the damping of such motions. This explanation has many lacunae. Previous photographic observations demonstrate that the secondary motions present near the free surface of the film do not penetrate to the wall (Oliver and Atherinos, 1968). Others (e.g., Kramers and Kreyger, 1956; Iribarne et al., 1967; Oliver and Atherinos, 1968) have shown that the solid-liquid mass transfer coefficient is not enhanced as a result of wavy motion. This cannot be explained if the disturbances emanating at the free surface were to reach the solid-liquid interface and disturb the diffusional boundary layer periodically. This implies that the fluid motion in the vicinity of the solid surface remains practically unaltered; therefore suppression of stretching motions will have hardly any effect on the film thickness.

It is well known that periodic mixing occurring near the free interface is responsible for enhancement in gas absorption rates (Oliver and Atherinos, 1967). If the secondary motions were to be damped as proposed by Popadic, there should be a reduction in the rate of gas absorption on polymer addition. Figure 1 presents the data from the work of Soylu (1974), who obtained gas absorption data in polymer by absorbing pure carbon dioxide in falling films. The ratio of liquid-phase mass transfer coefficient for a polymer solution to that for a solvent (in this case, water) is shown as a function of the Reynolds number Re. It is seen that under laminar rippling-flow conditions, the absorption rates actually increase thereby giving no evidence of suppression of wavy flow; if at all, the motion seems to be augmented.

In view of the evidence presented in the foregoing, we shall present an alternative hypothesis now and substantiate it by analyzing the data presented by Popadic.

SLIP HYPOTHESIS

Whenever macromolecular solutions are subjected to nonhomogeneous flows where the strain rate changes with position, there is a migration of macromolecules from high strain rate region to low strain rate region. Mechanistic explanations based on entropic

consideration have been presented previously (Tirrell and Malone, 1977; Metzner et al., 1979). As a result of such phenomena, a solvent layer almost completely depleted of polymer molecules is created in the neighborhood of solid surface. This gives rise to an apparent slip condition. Hydrodynamically, the problem could be treated by defining a finite slip velocity u_s at the solid wall. Some investigators of polymer solutions film flows (Astarita et al., 1964; Carreau et al., 1979) have detected such major slip effects in nonrippling laminar films. With concentrated polymer solutions, slip velocities which are as much as 80% of the mean bulk velocities have been observed (Metzner et al., 1979). We propose that the drag reduction phenomenon observed by Popadic is actually a slip effect.

THEORY

It is known that the wavy motion at the interface has a very small influence on the viscous drag (Popadic, 1974). This is clearly evident from the work of Tyler (1971) and Popadic (1975), where it is shown that for Newtonian fluids the experimental data fits the relationship $f = 24/Re_o$ reasonably well if δ is taken as the mean film thickness.

Thus, as a first approximation, we shall disregard the minor contribution to the viscous drag due to the wavy motion in the course of the subsequent analysis. Besides, since we are attempting to rationalize major differences in friction factors which are of the order of 50%, it appears that there is probably no necessity of introducing the complex effects due to the wave formation for the range of approximation invoked in the present work.

We assume that a power law fluid (consistency index, K, and power-law index, n) flows down a vertical plane. The slip boundary condition can be used to obtain an expression for the volumetric flow rate per unit width q as

$$q = u_s \delta + \left(\frac{\rho g}{K}\right)^{1/n} \left(\frac{n}{2n+1}\right) \delta^{2+1/n} \qquad (1)$$

where δ is the mean film thickness, ρ is the density, and g is the gravitational acceleration.
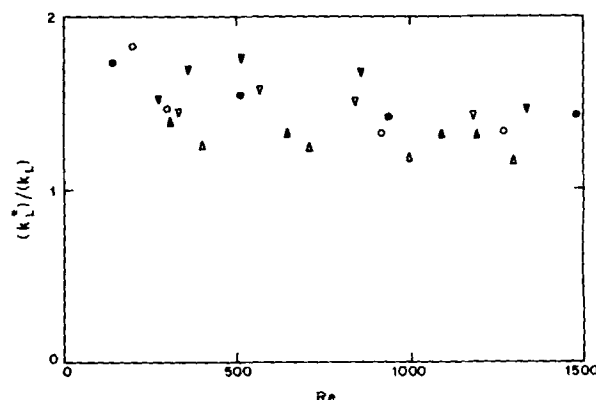

Figure 1. Ratio of mass transfer coefficients as a function of Reynolds number (Soylu, 1974). PEO [coag.]: (○) 0.02 (●) 0.05. PEO [WSR-301]: (▽) 0.02 (▼) 0.05 HEC: (△) 0.02 (▲) 0.05% in water.
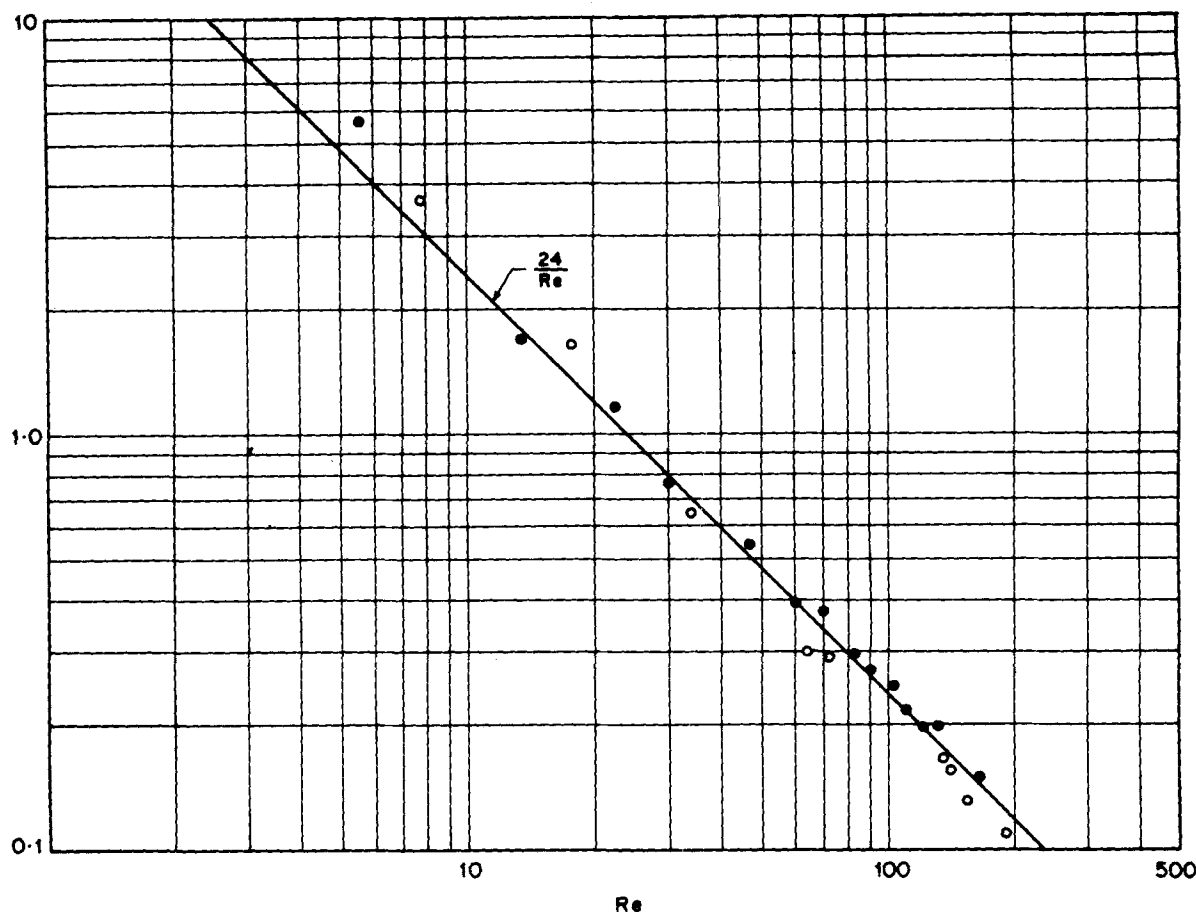
Figure 2. Friction factor as a function of Reynolds Number. Symbols: (O) 0.05% (●) 0.16% aqueous carbopol solution (Popadic, 1975).

Since the friction factor, $f = 2g\delta^3/q^2$, re-organization of Eq. 1 can be easily made to yield

$$f = \frac{24}{Re} \tag{2}$$

where

$$Re = Re_o + Re_s$$

with

$$Re_o = \frac{12n}{2n + 1}\left(\frac{q}{g\delta}\right)\left(\frac{\rho_g\delta}{K}\right)^{1/n} \tag{3}$$

and

$$Re_s = \frac{12qu_s}{g\delta^2} \tag{4}$$

Note that Re_s denotes the contribution due to the slip effect. Equation 3 immediately signifies that a finite value of Re_s resulting from slip will manifest itself in showing a reduction in f on conventional f-Re_o plots as used by Popadic.

We now assume that the slip velocity is linearly proportional to the wall shear stress $(T_w = \rho g\delta)$

$$u_s = \beta\rho g\delta \tag{5}$$

where β is the slip coefficient assumed to be constant. Equation 1 can be re-organized as

$$\frac{q}{\delta^2} = \beta\rho g + \left[\frac{\rho g}{K}\right]^{1/n}\left(\frac{n}{2n + 1}\right)\delta^{1/n} \tag{6}$$

Since β, ρ, g, K, and n are constants, a plot of q/δ^2 vs. $\delta^{1/n}$ should be linear. We used Popadic's data on 0.05% and 0.16% carbopol solutions and obtained β values of 0.1826 cm²·s/g and 0.03946 cm²·s/g, respectively, for these solutions. Observed reduction in β is consistent with the observation made by Metzner et al. (1979)

and Carreau et al. (1979). Using these β values, we have replotted the data in the traditional f-Re plots, Fig. 2. It is seen that the data fit Eq. 2 very well indeed.

In conclusion, we suggest that the unusual drag reduction phenomenon observed by Popadic in laminar rippling films of polymer solutions is actually a result of macromolecular migration away from the solid surface giving rise to an apparent slip effect and not due to its suppression. The data could be easily fitted in the new framework devised on the basis of Eq. 2. Note that, unlike Popadic's interpretation, the slip hypothesis also explains the mass transfer enhancement at the free interface (Figure 1) as observed by Soylu (1974). This augmentation is essentially a consequence of increased free-interface velocities resulting from effective fluid slippage at the solid boundary (Mashelkar and Dutta, 1982).

NOTATION

f = friction factor, dimensionless
g = acceleration due to gravity, cm/s²
k_L = mass transfer coefficient, cm/s
K = consistency index, dyne s^n/cm²
n = power-law index, dimensionless
q = flow rate per unit width, cm²/s
Re = Reynolds number, dimensionless
Re_o = Reynolds number as defined in Eq. 3, dimensionless
Re_s = Reynolds number due to slip, dimensionless slip velocity, cm/s

Greek Letters

β = slip coefficient, cm²·s/g
δ = mean film thickness, cm
ρ = density, g/cm³

LITERATURE CITED

Astarita, G., G. Marrucci, and G. Palumbo, "Non Newtonian Gravity Flow along Inclined Plane Surfaces," *Ind. Eng. Chem. Fund.*, **3**, 333 (1964).

Carreau, P. J., Q. H. Bui, and P. Leroux, "Wall Effects in Polymer Flow on Inclined Plane," *Rheol. Acta*, **18**, 600 (1979).

Iribarne, A., A. D. Gosman, and D. B. Spalding, "A Theoretical and Experimental Investigation of Diffusion—Controlled Electrolytic Mass Transfer between a Falling Liquid Film and a Wall," *Int. J. Heat Mass Tr.*, **10**, 1661 (1967).

Kale, D. D., R. A. Mashelkar, and J. J. Ulbrecht, "Drag Reduction in Rotational Viscoelastic Boundary Layer Flows," *Nature*, **29**, 242 (1973).

Kramers, H., and P. J. Kreyger, "Mass Transfer between a Flat Surface and a Falling Liquid Film," *Chem. Eng. Sci.*, **6**, 42 (1956).

Mashelkar, R. A., and G. V. Devrajan, "Secondary Flows of Non-Newtonian Fluids II: Frictional Losses in Laminar Flow of Viscoinelastic and Vis-

coelastic Fluids through Coiled Tubes," *Trans. Instn. Chem. Eng.*, **54**, 108 (1976).

Mashelkar, R. A., and A. Dutta, "Convective Diffusion in Structured Fluids: Need for New Analysis and Design Strategies," *Chem. Eng. Sci.* **37**, 969 (1982).

Metzner, A. B., Y. Cohen, and C. Rangel-Nafaile, "In homogeneous Flows of Non-Newtonian Fluids: Generation of Spatial Concentration Gradients," *J. Non-Newt. Fluid Mech.*, **5**, 449 (1979).

Oliver, D. R., and T. E. Atherinos, "Mass Transfer to Liquid Films on a Inclined Plane," *Chem. Eng. Sci.*, **23**, 525 (1968).

Popadic, V. O., "Drag Reduction in Film Flow," *AIChE J.*, **21**, 610 (1975).

Popadic, V. O., "Pseudoplastic Thin Film Flow," Ph.D. Dissertation, University of Kentucky, Lexington (1974).

Soylu, M., "Mass Transport Phenomena in Polymeric Solutions," Ph.D. Thesis, University of Salford, England (1974).

Tirrell, M., and M. F. Malone, "Macromolecules in Nonhomogeneous Velocity Gradient Fields," *J. Polym. Sci.*, **15**, 1569 (1977).

Tyler, J. S., "Non-Newtonian Thin Film Flow," Ph.D. Dissertation, University of Notre Dame, Indiana (1971).

Simple Method for Solution of a Class of Reaction-Diffusion Problems

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In reaction-diffusion systems, one is frequently faced with a physical situation the mathematical description of which requires the solution of nonlinear coupled differential equations. The nonlinearity in these equations exists either in the source term (e.g., reaction rate), in the derivative term (e.g., when volume changes exist), in the coefficients of the governing equations (concentration temperature dependencies of transport parameters), or in the boundary conditions.

It is possible in a given situation to have part or all of these nonlinearities present simultaneously. No analytical solution can be found in such cases and resort to numerical analysis has to be taken. Over the years, a number of papers have appeared that attempt to solve these problems with less effort in computation, and many of the methods proposed have been successful and are now widely used (Kubicek et al., 1970, 1971a,b,c; Hanna, 1980; Hall and Watt, 1976; Hornbeck, 1975; Weisz and Hicks, 1962; Finlayson, 1974).

The present paper is concerned with problems which typically involve nonlinearity in the source term and coupled boundary conditions. Such a physical situation is encountered in gas-solid catalytic reactions in pellets having bimodal pore structure. In gas-solid noncatalytic reaction, this situation arises when particle-pellet (grain) type of models are used to describe the behavior. Besides, in many biochemical systems involving encapsulated enzymes, such a situation is fairly common.

The problem thus appears to be fairly general, and we present below the methodology for solving such problems by considering

a specific example: calculation of the effectiveness factor for a catalyst pellet with bimodal pore distribution. The problem is the same as that considered by Ors and Dogu (1979), who have, however, restricted the solution to a linear reaction rate situation. Kulkarni et al. (1981) have also considered this problem, and for a pellet with regular geometry obtained an asymptotic expression for effectiveness factor for nonlinear rate terms. The use of the present method allows one to compute the effectiveness factor over the entire range of Thiele modulus with considerable saving in computation time.

The conservation equations in dimensionless form can be written as

$$\frac{d^2C_i}{dx^2} + \frac{2}{x}\frac{dC_i}{dx} = \phi^2[C_i]^n \tag{1}$$

$$\frac{d^2C_a}{dy^2} + \frac{2}{y}\frac{dC_a}{dy} = \alpha\left(\frac{dC_i}{dx}\right)_{x=1} \tag{2}$$

with the boundary conditions

$$x = 0, \quad \frac{dC_i}{dx} = 0 \tag{3}$$

$$x = 1, \quad C_i = C_a \tag{4}$$

$$y = 0 \quad \frac{dC_a}{dy} = 0 \tag{5}$$

$$y = 1 \quad C_a = 1 \tag{6}$$

where the various dimensionless terms are defined in the notation.